RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2019

FIRST YEAR [BATCH 2018-21]

Date : 27/05/2019 **STATISTICS (General)**

Time : 11 am – 1 pm **Paper : II** Full Marks : 50

[Use a separate Answer Book for each group]

Group - A

1. Answer **any two** questions : [2×5]

a) What do you mean by multiple correlation coefficient? Taking three variables, express multiple correlation coefficient in terms of simple correlation coefficient. (2+3)

b) Show that $\left(1-r_{1.23}^2\right)=\left(1-r_{12}^2\right)\left(1-r_{13.2}^2\right)$ where symbols have their usual meaning. Hence or otherwise prove that the value of multiple correlation coefficient cannot be less than the value of any total correlation coefficient or the value of any partial correlation coefficient. (2.5+2.5)

c) Suppose the three variables x_1,x_2 and x_3 satisfy the relation $a_1x_1+a_2x_2+a_3x_3=K$, then prove that $r_{12}=\dfrac{a_3s_3^2-a_1s_1^2-a_2s_2^2}{2a_1a_2s_1s_2}$

Also prove that all the partial correlation coefficient will be equal to -1, provided constants a_1,a_2,a_3 are of same sign. (3+2)

d) Define Correlation index. Show that the value of correlation index increases with degree of the polynomial taken as regression equation. (2+3)

2. Answer **any one** question : [1×15]

a) If r and e_{yx} are respectively correlation coefficient and correlation ration the prove that $r^2 \le e_{yx}^2 \le 1$ interpret $e_{yx}^2=r^2$ and $r^2<e_{yx}^2=1$ In which aspect partial correlation is different from multiple correlation? (5+(3+3)+4)

b) Define attribute? What is contingency table? State three measures of association of attributes with their ranges. Write your idea about binary response and logistic regression. (3+3+6+3)

Group - B

3. Answer **any two** questions : [2×5]

a) In an oil exploration in the Arabian sea, suppose that the probability of an oil strike is 1 in 500 drillings. What is the probability of exactly one oil producing well in 800 explorations? (5)

b) Suppose you are playing a game of throwing darts at a board with your friend. He will win the game who can hit the bull’s eye at the centre of the board thrice. Your chance of hitting the bull’s eye is 0.6 and this remains constant from throw to throw. Find the probability that you need 5 throws to win the game. (5)

c) Prove the memoryless property of geometric distribution. (5)

d) Derive the expression of mean deviation about mean of the variable which follows normal distribution with mean μ and variance σ^2. (5)

4. Answer **any one** question : [1×15]

a) i) State the central limit theorem. (3)

ii) Show, in the context of bivariate Normal distribution, the value of correlation coefficient as zero implies the independence of two random variables. (3)

iii) Let X ~ Binomial (n, p). Find out the value of p for which the variance of X maximum. (3)

iv) Use the Normal approximation to the binomial distribution to determine the probability that number of heads lies between 6 & 8 in 16 flips of a balanced coin. (6)

b) i) A person selects a number randomly from the set {1,2,3,4,5}. Next he draws another number from the set of integers remaining after discarding all integers less than the first selected integer. Let X & Y denote the numbers drawn in the 1st & 2nd draw respectively. Construct a bivariate table showing the joint probability distribution of X & Y. Also find P (X+Y > 7). (7)

ii) If X ~ Normal (0, 1), find the distribution of e^X and its mean. (5)

iii) State the Weak law of Large numbers. (3)

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TABLE I ORDINATES AND AREAS OF THE DISTRIBUTION OF STANDARD NORMAL VARIABLE*

τ	φ(τ)	Φ(τ)	τ	φ(τ)	Φ(τ)	τ	φ(τ)	Φ(τ)
·00	·3989423	·5000000						
·01	·3989223	·5039894	·51	·3502919	·6949743	1·01	·2395511	·8437524
·02	·3988625	·5079783	·52	·3484925	·6984682	1·02	·2371320	·8461358
·03	·3987628	·5119665	·53	·3466677	·7019440	1·03	·2347138	·8484950
·04	·3986233	·5159534	·54	·3448180	·7054015	1·04	·2322970	·8508300
·05	·3984439	·5199388	·55	·3429439	·7088403	1·05	·2298821	·8531409
·06	·3982248	·5239222	·56	·3410458	·7122603	1·06	·2274696	·8554277
·07	·3979661	·5279032	·57	·3391243	·7156612	1·07	·2250599	·8576903
·08	·3976677	·5318814	·58	·3371799	·7190427	1·08	·2226535	·8599289
·09	·3973298	·5358564	·59	·3352132	·7224047	1·09	·2202508	·8621434
·10	·3969525	·5398278	·60	·3332246	·7257469	1·10	·2178522	·8643339
·11	·3965360	·5437953	·61	·3312147	·7290691	1·11	·2154582	·8665005
·12	·3960802	·5477584	·62	·3291840	·7323711	1·12	·2130691	·8686431
·13	·3955854	·5517168	·63	·3271330	·7356527	1·13	·2106856	·8707619
·14	·3950517	·5556700	·64	·3250623	·7389137	1·14	·2083078	·8728568
·15	·3944793	·5596177	·65	·3229724	·7421539	1·15	·2059363	·8749281
·16	·3938684	·5635595	·66	·3208638	·7453731	1·16	·2035714	·8769756
·17	·3932190	·5674949	·67	·3187371	·7485711	1·17	·2012135	·8789995
·18	·3925315	·5714237	·68	·3165929	·7517478	1·18	·1988631	·8809999
·19	·3918060	·5753454	·69	·3144317	·7549029	1·19	·1965205	·8829768
·20	·3910427	·5792597	·70	·3122539	·7580363	1·20	·1941861	·8849303
·21	·3902419	·5831662	·71	·3100603	·7611479	1·21	·1918602	·8863606
·22	·3894038	·5870644	·72	·3078513	·7642375	1·22	·1895432	·8887676
·23	·3885286	·5909541	·73	·3056274	·7673049	1·23	·1872354	·8906514
·24	·3876166	·5948349	·74	·3033893	·7703500	1·24	·1849373	·8925123
·25	·3866681	·5987063	·75	·3011374	·7733726	1·25	·1826491	·8943502
·26	·3856834	·6025681	·76	·2988724	·7763727	1·26	·1803712	·8961653
·27	·3846627	·6064199	·77	·2965948	·7793501	1·27	·1781038	·8979577
·28	·3836063	·6102612	·78	·2943050	·7823046	1·28	·1758474	·8997274
·29	·3825146	·6140919	·79	·2920038	·7852361	1·29	·1736022	·9014747
·30	·3813878	·6179114	·80	·2896916	·7881446	1·30	·1713686	·9031995
·31	·3802264	·6217195	·81	·2873689	·7910299	1·31	·1691468	·9049021
·32	·3790305	·6255158	·82	·2850364	·7938919	1·32	·1669370	·9065825
·33	·3778007	·6293000	·83	·2826945	·7967306	1·33	·1647397	·9082409
·34	·3765372	·6330717	·84	·2803438	·7995458	1·34	·1625551	·9098773
·35	·3752403	·6368307	·85	·2779849	·8023375	1·35	·1603833	·9114920
·36	·3739106	·6405764	·86	·2756182	·8051055	1·36	·1582248	·9130850
·37	·3725483	·6443088	·87	·2732444	·8078498	1·37	·1560797	·9146565
·38	·3711539	·6480273	·88	·2708640	·8105703	1·38	·1539483	·9162067
·39	·3697277	·6517317	·89	·2684774	·8132671	1·39	·1518308	·9177356
·40	·3682701	·6554217	·90	·2660852	·8159399	1·40	·1497275	·9192433
·41	·3667817	·6590970	·91	·2636880	·8185887	1·41	·1476385	·9207302
·42	·3652627	·6627573	·92	·2612863	·8212136	1·42	·1455641	·9221962
·43	·3637136	·6664022	·93	·2588805	·8238145	1·43	·1435046	·9236415
·44	·3621349	·6700314	·94	·2564713	·8263912	1·44	·1414600	·9250663
·45	·3605270	·6736448	·95	·2540591	·8289439	1·45	·1394306	·9264707
·46	·3588903	·6772419	·96	·2516443	·8314724	1·46	·1374165	·9278550
·47	·3572253	·6808225	·97	·2492277	·8339768	1·47	·1354181	·9292191
·48	·3555325	·6843863	·98	·2468095	·8364569	1·48	·1334353	·9305634
·49	·3538124	·6879331	·99	·2443904	·8389129	1·49	·1314684	·9318879
·50	·3520653	·6914625	1·00	·2419707	·8413447	1·50	·1295176	·9331928

TABLE I (*Contd.*)

τ	φ(τ)	Φ(τ)	τ	φ(τ)	Φ(τ)	τ	φ(τ)	Φ(τ)
1·51	·1275830	·9344783	2·01	·0529192	·9777844	2·51	·0170947	·9939634
1·52	·1256646	·9357445	2·02	·0518636	·9783083	2·52	·0166701	·9941323
1·53	·1237628	·9369916	2·03	·0508239	·9788217	2·53	·0162545	·9942969
1·54	·1218775	·9382198	2·04	·0498001	·9793248	2·54	·0158476	·9944574
1·55	·1200090	·9394292	2·05	·0487920	·9798178	2·55	·0154493	·9946139
1·56	·1181573	·9406201	2·06	·0477996	·9803007	2·56	·0150596	·9947664
1·57	·1163225	·9417924	2·07	·0468226	·9807738	2·57	·0146782	·9949151
1·58	·1145048	·9429466	2·08	·0458611	·9812372	2·58	·0143051	·9950600
1·59	·1127042	·9440826	2·09	·0449148	·9816911	2·59	·0139401	·9952012
1·60	·1109208	·9452007	2·10	·0439836	·9821356	2·60	·0135830	·9953388
1·61	·1091548	·9463011	2·11	·0430674	·9825708	2·61	·0132337	·9954729
1·62	·1074061	·9473839	2·12	·0421661	·9829970	2·62	·0128921	·9956035
1·63	·1056748	·9484493	2·13	·0412795	·9834142	2·63	·0125581	·9957308
1·64	·1039611	·9494974	2·14	·0404076	·9838226	2·64	·0122315	·9958547
1·65	·1022649	·9505285	2·15	·0395500	·9842224	2·65	·0119122	·9959754
1·66	·1005864	·9515428	2·16	·0387069	·9846137	2·66	·0116001	·9960930
1·67	·0989255	·9525403	2·17	·0378779	·9849966	2·67	·0112951	·9962074
1·68	·0972823	·9535213	2·18	·0370629	·9853713	2·68	·0109969	·9963189
1·69	·0956568	·9544860	2·19	·0362619	·9857379	2·69	·0107056	·9964274
1·70	·0940491	·9554345	2·20	·0354746	·9860966	2·70	·0104209	·9965330
1·71	·0924591	·9563671	2·21	·0347009	·9864474	2·71	·0101428	·9966358
1·72	·0908870	·9572838	2·22	·0339408	·9867906	2·72	·0098712	·9967359
1·73	·0893326	·9581849	2·23	·0331939	·9871263	2·73	·0096058	·9968333
1·74	·0877961	·9590705	2·24	·0324603	·9874545	2·74	·0093466	·9969280
1·75	·0862773	·9599408	2·25	·0317397	·9877755	2·75	·0090936	·9970202
1·76	·0847764	·9607961	2·26	·0310319	·9880894	2·76	·0088465	·9971099
1·77	·0832932	·9616364	2·27	·0303370	·9883962	2·77	·0086052	·9971972
1·78	·0818278	·9624620	2·28	·0296546	·9886962	2·78	·0083697	·9972821
1·79	·0803801	·9632730	2·29	·0289847	·9889893	2·79	·0081398	·9973646
1·80	·0789502	·9640697	2·30	·0283270	·9892759	2·80	·0079155	·9974449
1·81	·0775379	·9648521	2·31	·0276816	·9895559	2·81	·0076965	·9975229
1·82	·0761433	·9656205	2·32	·0270481	·9898296	2·82	·0074829	·9975988
1·83	·0747663	·9663750	2·33	·0264265	·9900969	2·83	·0072744	·9976726
1·84	·0734068	·9671159	2·34	·0258166	·9903581	2·84	·0070711	·9977443
1·85	·0720649	·9678432	2·35	·0252182	·9906133	2·85	·0068728	·9978140
1·86	·0707404	·9685572	2·36	·0246313	·9908625	2·86	·0066793	·9978818
1·87	·0694333	·9692581	2·37	·0240556	·9911060	2·87	·0064907	·9979476
1·88	·0681436	·9699460	2·38	·0234910	·9913437	2·88	·0063067	·9980116
1·89	·0668711	·9706210	2·39	·0229374	·9915758	2·89	·0061274	·9980738
1·90	·0656158	·9712834	2·40	·0223945	·9918025	2·90	·0059525	·9981342
1·91	·0643777	·9719334	2·41	·0218624	·9920237	2·91	·0057821	·9981929
1·92	·0631566	·9725711	2·42	·0213407	·9922397	2·92	·0056160	·9982498
1·93	·0619524	·9731966	2·43	·0208294	·9924506	2·93	·0054541	·9983052
1·94	·0607652	·9738102	2·44	·0203284	·9926564	2·94	·0052963	·9983589
1·95	·0595947	·9744119	2·45	·0198374	·9928572	2·95	·0051426	·9984111
1·96	·0584409	·9750021	2·46	·0193563	·9930531	2·96	·0049929	·9984618
1·97	·0573038	·9755808	2·47	·0188850	·9932443	2·97	·0048470	·9985110
1·98	·0561831	·9761482	2·48	·0184233	·9934309	2·98	·0047050	·9985588
1·99	·0550789	·9767045	2·49	·0179711	·9936128	2·99	·0045666	·9986051
2·00	·0539910	·9772499	2·50	·0175283	·9937903	3·00	·0044318	·9986501

TABLE I (*Contd.*)

τ	$\phi(\tau)$	$\Phi(\tau)$	τ	$\phi(\tau)$	$\Phi(\tau)$	τ	$\phi(\tau)$	$\Phi(\tau)$
3·01	·0043007	·9986938	3·21	·0023089	·9993363	3·41	·0011910	·9996752
3·02	·0041729	·9987361	3·22	·0022358	·9993590	3·42	·0011510	·9996869
3·03	·0040486	·9987772	3·23	·0021649	·9993810	3·43	·0011122	·9996982
3·04	·0039276	·9988171	3·24	·0020960	·9994024	3·44	·0010747	·9997091
3·05	·0038098	·9988558	3·25	·0020290	·9994230	3·45	·0010383	·9997197
3·06	·0036951	·9988933	3·26	·0019641	·9994429	3·46	·0010030	·9997299
3·07	·0035836	·9989297	3·27	·0019010	·9994623	3·47	·0009689	·9997398
3·08	·0034751	·9989650	3·28	·0018397	·9994810	3·48	·0009358	·9997493
3·09	·0033695	·9989992	3·29	·0017803	·9994991	3·49	·0009037	·9997585
3·10	·0032668	·9990324	3·30	·0017226	·9995166	3·50	·0008727	·9997674
3·11	·0031669	·9990646	3·31	·0016666	·9995335	3·51	·0008426	·9997759
3·12	·0030698	·9990957	3·32	·0016122	·9995499	3·52	·0008135	·9997842
3·13	·0029754	·9991260	3·33	·0015595	·9995658	3·53	·0007853	·9997922
3·14	·0028835	·9991553	3·34	·0015084	·9995811	3·54	·0007581	·9997999
3·15	·0027943	·9991836	3·35	·0014587	·9995959	3·55	·0007317	·9998074
3·16	·0027075	·9992112	3·36	·0014106	·9996103	3·56	·0007001	·9998146
3·17	·0026231	·9992378	3·37	·0013639	·9996242	3·57	·0006814	·9998215
3·18	·0025412	·9992636	3·38	·0013187	·9996376	3·58	·0006575	·9998282
3·19	·0024615	·9992886	3·39	·0012748	·9996505	3·59	·0006343	·9998347
3·20	·0023841	·9993129	3·40	·0012322	·9996631	3·60	·0006119	·9998409

***Abridged from Table 1 of *Biometrika Tables for Statisticians*, vol. I, with the kind permission of the Biometrika Trustees.**

TABLE II STANDARD NORMAL DISTRIBUTION

Values of τ_α

α	0·05	0·025	0·01	0·005
τ_α	1·645	1·960	2·326	2·576